Paper Id: 110301 Roll No: | | | | | | | | | | | | | |

B. TECH.
(SEM-III) THEORY EXAMINATION 2019-20
DISCRETE STRUCTURES & THEORY OF LOGIC

Time: 3 Hours *Total Marks: 70*

Note: **1.** Attempt all Sections. If require any missing data; then choose suitably.
2. Any special paper specific instruction.

SECTION A

1. Attempt *all* questions in brief. **2 x 7 = 14**

a. Draw all trees with exactly six vertices.

b. Find the adjacency matrix A = [a_{ij}] of graph given in figure 1.

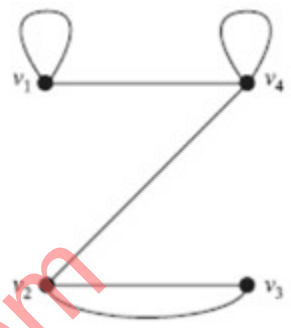

Figure 1

c. Determine the power set P(A) of A = {a, b, c, d}.

d. Define surjective function.

e. Let f and g be the functions from the set of integers to the set of integers defined by f (x) = 2x + 3 and g(x) = 3x + 2. What is the composition of f and g? What is the composition of g and f?

f. Consider the following relations on {1, 2, 3, 4}:
R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},
R2 = {(1, 1), (1, 2), (2, 1)},
R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},
R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},
R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},
R6 = {(3, 4)}.
Which of these relations are reflexive?

g. List all the ordered pairs in the relationR = {(a, b) | a divides b} on the set {1, 2, 3, 4, 5, 6} and also display the graphical representation of the same.

h. Find the values of the Boolean function represented by F (x, y, z) = xy + z'.

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

a. Prove the proposition *P(n)* that the sum of the first *n* positive integers is $n(n+1)$; that is, $P(n) = 1 + 2 + 3 + \cdots + n = n(n+1)$

b. Verify that the given propositionsare tautology or not.
i. $p \vee \neg (p \wedge q)$
ii. $\neg p \wedge q$

c. Prerequisites in college is a familiar partial ordering of available classes. We write $A \prec B$ if course *A* is a prerequisite for course *B*. Let *C* be the ordered set consisting of the mathematics courses and theirprerequisites appearing in figure 2.

Class	Prerequisites
Math 101	None
Math 201	Math 101
Math 250	Math 101
Math 251	Math 250
Math 340	Math 201
Math 341	Math 340
Math 450	Math 201, Math 250
Math 500	Math 450, Math 251

Figure 2

i. Draw the Hasse diagram for the partial ordering *C* of these classes.
ii. Find all minimal and maximal elements of *C*.
iii. Does *C* have a first element or a last element?

Paper Id: 110301 Roll No:

d. What are the degrees and what are the neighborhoods of the vertices in the graphs G and H displayed in Figure 3?

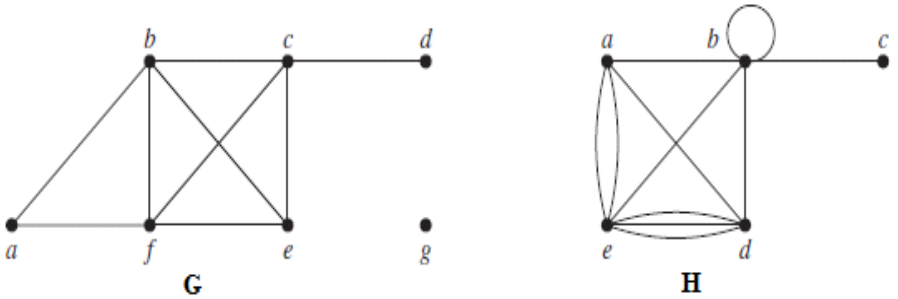

Figure 3

e. For which values of n do these graphs have an Euler path but no Euler circuit?
i. K_n ii.C_n iii. W_n iv.Q_n

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

(a) Answer these questions for the poset$(\{3, 5, 9, 15,24, 45\}, |)$.
i. Find the maximal elements. ii. Find the minimal elements.
iii. Is there a greatest element? iv. Is there a least element?
v. Find all upper bounds of {3, 5}.vi. Find the least upper bound of {3, 5}.
vii. Find all lower bounds of {15, 45}. viii.Find the greatest lower bound of {15, 45}, if it exists.

(b) Are the graphs G and H displayed in Figure 4 bipartite?

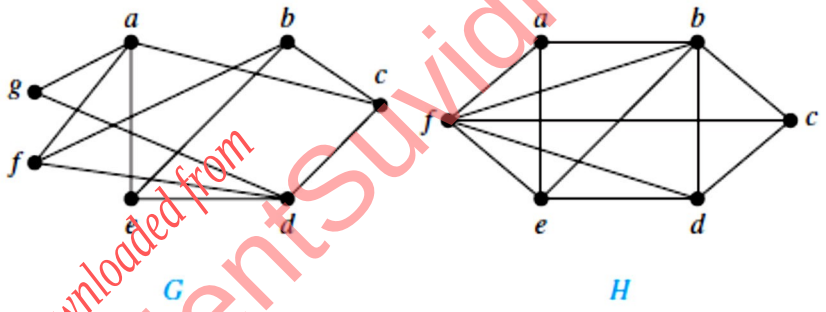

Figure 4

4. Attempt any *one* part of the following: 7 x 1 = 7

(a) Represent the expressions $(x + xy) + (x/y)$ and$x + ((xy + x)/y)$ using binary trees.Write these expressions in:
i. prefix notation.
ii. postfix notation.
iii. infix notation.

(b) Construct the ordered rooted tree whose preorder traversalis *a, b, f, c, g, h, i, d, e, j, k, l*, where *a* has four children,*c* has three children, *j* has two children, *b* and *e*have onechild each, and all other vertices are leaves.

5. Attempt any *one* part of the following: 7 x 1 = 7

(a) Determine whether each of these statements is true orfalse.
i) $0 \in \emptyset$ **ii)** $\emptyset \in \{0\}$
iii) $\{0\} \subset \emptyset$ iv) $\emptyset \subset \{0\}$
v) $\{0\} \in \{0\}$ vi)$\{0\} \subset \{0\}$
vii) $\{\emptyset\} \subseteq \{\emptyset\}$

(b) For each of these relations on the set {1, 2, 3, 4}, decidewhether it is reflexive, whether it is symmetric, whetherit is antisymmetric, and whether it is transitive.
i) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
ii) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
iii) {(2, 4), (4, 2)}
iv) {(1, 2), (2, 3), (3, 4)}
v) {(1, 1), (2, 2), (3, 3), (4, 4)}
vi) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

6. Attempt any *one* part of the following: 7 x 1 = 7

(a) Which of the partially ordered sets in Fig 5 are lattices?

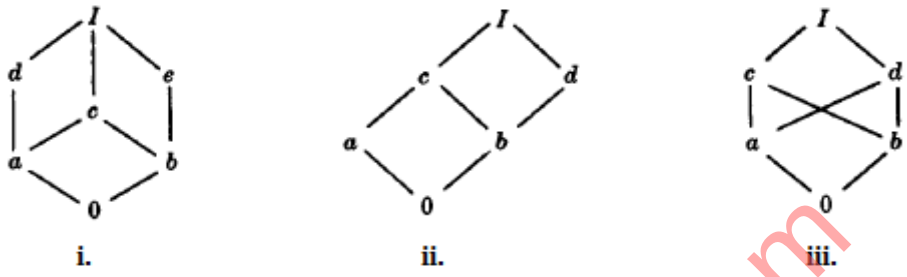

Figure5

(b) What are the chromatic numbers of the graphs *G* and *H* shown in Figure 6

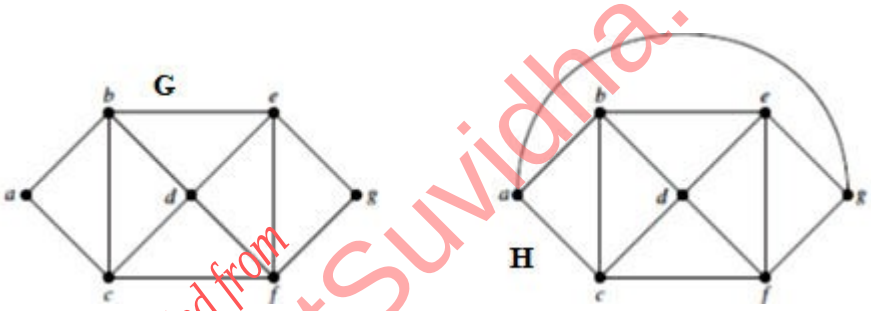

Figure6

7. Attempt any *one* part of the following: 7 x 1 = 7

(a) Consider the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7.
(*i*) Find the multiplication table of *G*. (*ii*) Find 2^{-1}, 3^{-1}, 6^{-1}.
(iii) Find the orders and subgroups generated by 2 and 3. (*iv*) Is *G* cyclic?

(b) How many cards must be selected from a standard deck of 52 cards to guarantee that at leastthree cards of the same suit are chosen?